Some Math Competition Tips and Tricks

1. **Number theory**
   a. **Know divisibility rules (3,4,6,9)**

   b. **Know perfect squares by heart up to 20^2**

11	12	13	14	15	16	17	18	19	20
121	144	169	196	225	256	289	324	361	400

   c. **Multiplication tricks**

   Ex 99x101 from $(a+b)(a-b) = a^2 - b^2$
   = (100-1)(100+1)
   = 10000-1
   =9999

   Try 52X48 *Try 33x27* *Try 999x1001*

   d. **Know powers of 2 up to 2^{10}** **and Primes less than 100**

   $2^2 = 4$
   $2^3 = 8$
   $2^4 = 2^2 \times 2^2 = 16$
   $2^5 = 32$
   $2^6 = 2^3 x 2^3 = 64$
   $2^7 = 128$
   $2^8 = 2^4 \times 2^4 = 256$
   $2^9 = 512$
   $2^{10} = 1024$

2	23	53	83
3	29	59	89
5	31	61	97
7	37	67	
11	41	71	
13	43	73	
17	47	79	
19			

   e. **11 times tables**

   Ex 54x11= 594 64x11= 704
   23x11= 253 56x11= 616
   81x11= 891 95x11= 1045

   f. **15 times tables**

   Ex 46x15= 690 38x15= 570
   24x15= 360 76x15= 1140
   66x15= 990 41x15= 615

2. Probability – Cards, Dice, Heads or tails

a. Cards (52 cards in a deck with 4 suits of 13 cards each)

Ex. What is the probability of drawing 2 clubs in a row without replacement?

P(2 clubs)= $= \left(\frac{13}{52}\right)\left(\frac{12}{51}\right)$

Try Probability of drawing 3 face cards in a row with replacement after each draw.

b. Dice

Ex. What is the probability of rolling two dice together resulting in a sum of 7?

	1	**2**	**3**	**4**	**5**	**6**
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

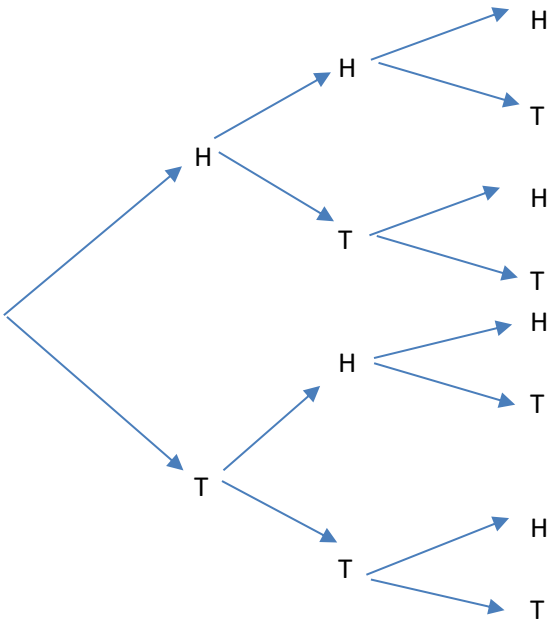
Note: How many of each outcome exists….there is an obvious pattern!

P(7) = $\frac{6}{36} = \frac{1}{6}$

Try probability of a total sum of 4 or higher? *(Hint: Look at P(<4))*

c. Heads or Tails

Ex. What is the probability of getting 2 heads and one tails in 3 coin tosses?



P(2H,1T)= $\frac{3}{8}$

Try P(No tails in 3 tosses)

d. Factorials

ex 5!=5x4x3x2x1=120

Ex $\frac{10!-9!-8!}{10!+9!+8!}=\frac{(8!)(10\times9-9-1)}{(8!)(10\times9+9+1)}=\frac{80}{100}=\frac{4}{5}$

Try $\frac{8!}{6!}$

Ex What is the probability of winning the 6-49 whereby you randomly select the 6 numbers out of the 49 without replacement?

P(6-49)= $\frac{6}{49}\times\frac{5}{48}\times\frac{4}{47}\times\frac{3}{46}\times\frac{2}{45}\times\frac{1}{44}=\frac{6!}{(49!\div43!)}$ But remember to reduce!

Ex. How many ways can the 4 letters "POLL" be arranged into a unique word?

Ans. $\frac{4!}{2}$

3. Fractions and percentages

Ex What is 20% of 20%? (ans 4%)

Ex How many apples did you start with if you finish with one apple after giving half your apples to your younger brother, and then half of your apples to your friend, and then half of the remaining apples to your horse.

1X2x2x2=8

Ex Fifty percent of fifty percent is 50. What is the number?

Ex. What is 4% of 75? (Hint and Cool Trick: What is 75% of 4?)

4. Binary Numbers

Binary numbers means that only 0 and 1 may be used. You are used to base 10, but computers and many other devices only use binary. A "0" means no and a "1" means yes.

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2^{10} 2^9 2^8 2^7 2^6 2^5 2^4 2^3 2^2 2^1 2^0

Ex Convert the following into binary. Start at the largest power of 2 that will fit in the number and continue down until the number is reached.

140 1 0 0 0 1 1 0 0 =128+8+4

2^{10} 2^9 2^8 2^7 2^6 2^5 2^4 2^3 2^2 2^1 2^0

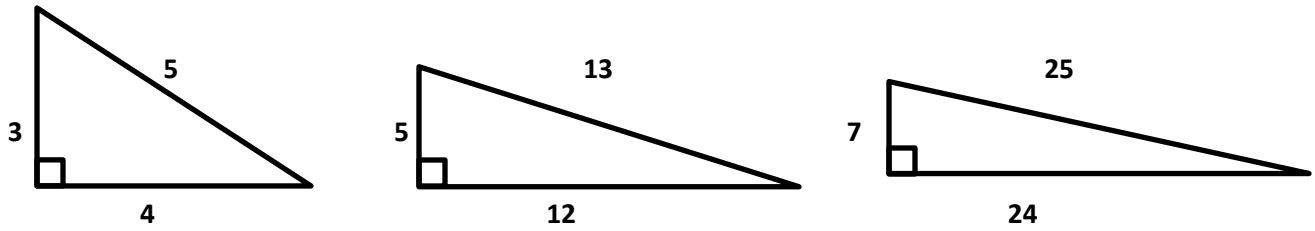
Try 5 in binary

Try what is one more than 11111 expressed in decimal?

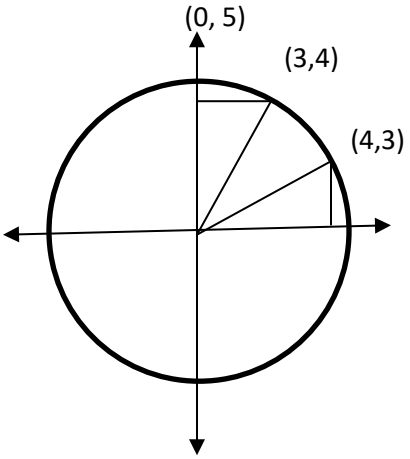
Try adding 24 and 15 in binary. Once you get your total, convert back to decimal to check it.

5. Standard triangles with applications

a. Standard length triangles

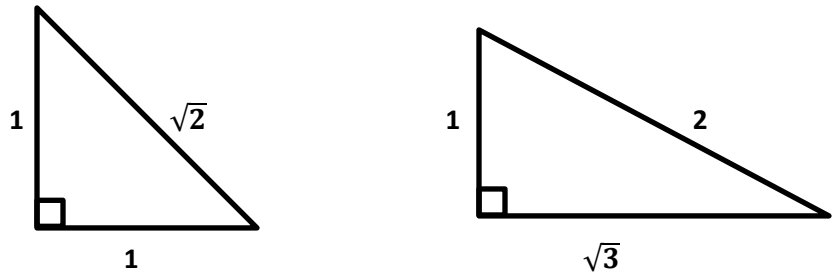


Ex A circle is drawn on an x, y axis system with a radius of 5. How many coordinate points on the circle have exact integer values?



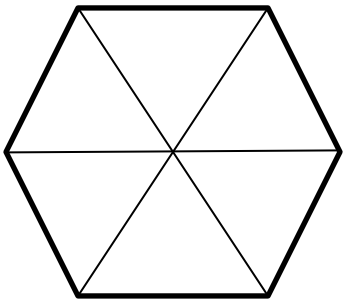
Ans. 12 (3 per quadrant)

b. Standard angle Triangles



Note: These "lengths" are NOT REALLY LENGTHS! They are just the proportionate length of each side given these angles. This allows for very quick calculations if an actual length of one of the sides of the triangle IS KNOWN!

6. Diagonals of a Polygon



Formula for number of diagonals of an "n" sided polygon:

$$\frac{n(n-3)}{2}$$

7. Relative Motion

Ex A boat can travel at 25km/h in still water. How long will it take to travel upstream 60 km if the current flows at 5 km/h? Ans. NET speed in this case is 25km/h – 5 km/h = 20km/h

Hence $\frac{60km}{\frac{20km}{h}} = 3\ hours$

8. Series and Sequences

Ex $x = 1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} - \frac{1}{32} + \frac{1}{64} - \frac{1}{128}$. What is the value of 128x? (It asks for the numerator!)

Ex 1+2+3+4+…+98+99+100=?

Ex $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots =$?

Arithmetic Sequence (Term) $t_n = a + (n-1)d$

Arithmetic sum $S_n = \frac{n}{2}(a + t_n)$ or $S_n = \frac{n}{2}[2a + (n-1)d]$

Geometric Sequence (Term) $t_n = ar^{n-1}$

Geometric finite Series (Total Sum) $S_n = \frac{a(1-r^n)}{1-r}$

Geometric Infinite Series (Total Sum) $S_\infty = \frac{a}{1-r}$

9. Pascal's Triangle

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

-Useful for many things, including powers of 11 (first line= 11^0 , second line=11^1 , third line=11^2 , etc.

-Also useful for $(a+b)^4 = 1a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + 1b^4$ for example (5th line for 4th power).

10. The Year of the Competition!

Factor 2023.	**Ans.** $\mathbf{7}, \mathbf{17}, \mathbf{17}$
Add the factors of 2023.	1 + 7 + 17 + 119 + 289 + 2023 = 2456
Sum of prime factors?	7+17 = 24

Ex. 2023 -is NOT a prime number (2017 is the previous one and 2027 is the next)
-2023_{10} when converted to binary is $\mathbf{11\ 111\ 100\ 111_2}$ is 2023_{10} in binary
-Nearest perfect square to 2023? $(45^2 = 2025)$